

Pure Maths With Mechs 3
0765

GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

JUNE 2024

ADVANCED LEVEL

Subject Title	Pure Mathematics with Mechanics
Paper No.	Paper 3
Subject Code No.	0765

Duration: Three Hours

Full marks may be obtained for answers to ALL questions.

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae booklet published by Board are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. The position vector $\mathbf{r}$ of a particle at time t is given by

$$\mathbf{r} = [(2t + 3)\mathbf{i} + (t^2 - 1)\mathbf{j}] \text{ m}.$$

Find the

- average velocity over the interval $t = 0$ to $t = 3$ s,
- speed and distance of the particle from the origin when $t = 1$ s,
- Cartesian equation of its path.

(5 marks)

(6 marks)

(2 marks)

2. A car travels on a straight horizontal track. The car decelerates uniformly from a speed of 20 m s^{-1} to a speed of 12 m s^{-1} as it travels a distance of 640 m. The car then accelerates uniformly, travelling a further 1820 m in 70 s. Calculate the

- time that it takes the car to travel the first 640 m,
- deceleration of the car during the first 640 m,
- acceleration of the car as it travels the further 1820 m,
- speed of the car when it has completed the further 1820 m,
- average speed of the car as it travels the 2460 m.

(3 marks)

(3 marks)

(3 marks)

(2 marks)

(2 marks)

3. Two Particles P and Q , of masses 6 kg and 4 kg respectively, are connected by a light inextensible string of length 2 m. The string passes over a light smooth pulley fixed at the top of a smooth plane which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. Initially, the particles are held at rest with the string taut, P lying on the plane and Q hanging over the pulley. The particles are then released from rest.

- Find the magnitude of the acceleration of the particles and the tension in the string.

(6 marks)

Given that particle Q is initially 1.5 m above the ground, and particle P does not reach the top of the plane. Calculate the

- speed which particle Q hits the ground,
- time that elapses between when Q reaches the ground and when the string becomes taut again.

(3 marks)

(4 marks)

(Take g as 10 m s^{-2} .)

4. Particles A and B , of masses 6 kg and 2 kg respectively, collide directly. The speeds of A and B after collision are 4 m s^{-1} and 6 m s^{-1} respectively and the coefficient of restitution between A and B is $\frac{1}{3}$.

- Calculate the initial velocities of A and B given that they were moving in the same direction.

(6 marks)

B moves and hits directly another stationary particle C of mass m kg, causing C to gain a speed of 10 m s^{-1} . Find

- m ,
- the kinetic energy lost by the system.

(4 marks)

(3 marks)

5. (i) A uniform ladder AB , of weight 40 N and length 8 m, is resting with A against a smooth vertical wall and B on rough horizontal ground. The coefficient of friction between the ladder and the ground is $\frac{1}{4}$. The ladder makes an angle θ with the horizontal ground, where $\tan \theta = \frac{3}{4}$. Given that, towards the wall, a horizontal force of magnitude 125 N is applied at a point D on the ladder, where $BD = 1.5$ m, find how far up the ladder a man of weight 200 N will climb before the ladder will begin to slip.

(8 marks)

- (ii) An elastic string AB of natural length 1.5 m is hanging from a fixed point A . A mass of 5 kg is suspended from the end B and produces an extension of 3 mm. calculate the

- modulus of elasticity of the string,
- elastic potential energy of the string.

(2 marks)

(3 marks)

(Take g as 10 m s^{-2} .)



6. (i) A car of mass 750 kg is moving up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{15}$. The resistance to the motion of the car from non-gravitational forces has a constant magnitude of R newtons. The power developed by the car's engine is 15 kW and the car is moving at a constant speed of 20 m/s.

(a) Find R .

(4 marks)

The power developed by the engine of the car is increased to 18 kW and the non-gravitational resistance remains the same, when the car is moving up a road at a speed of 20 m s⁻¹.

(b) Find its acceleration.

(4 marks)

- (ii) A bead is threaded on a straight wire. The vector equation of the wire is,

$$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \text{ m.}$$

The bead is moved from point A on the wire through a distance of 6 m along the wire to a point B by a force $\mathbf{F} = (7\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ N. Find the magnitude of the work done by $\mathbf{F}$ in moving the bead from A to B .

(5 marks)

(Take g as 10 m s⁻²)

7. (i) A trapezium $ABCD$ with uniform density is such that $AB = 4$ m and $BC = CD = DA = 2$ m.

(a) Show that the centre of mass of the lamina is $\frac{4\sqrt{3}}{9}$ m from AB .

(6 marks)

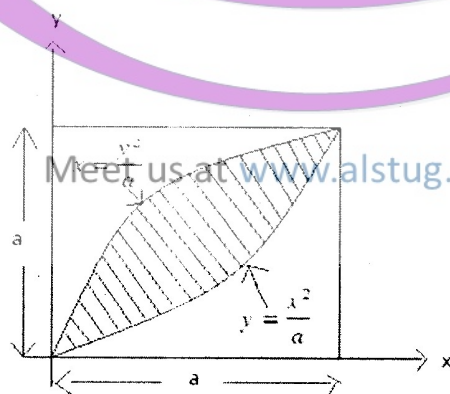
The lamina is freely suspended from D and hangs in equilibrium.

(b) Find the angle between DC and the vertical through D .

(3 marks)

- (ii) Given that the shaded area in figure below is bounded by two curves given by

$$x = \frac{y^2}{a} \text{ and } y = \frac{x^2}{a}.$$



- (c) Show, by integration, that the x - coordinate of the centroid of the shaded area is $\frac{9}{20}a$.

(4 marks)



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8. (i) Two events A and B are such that $P(A) = \frac{3}{10}$, and $P(A \cup B) = \frac{4}{5}$. Find,
(a) $P(B)$ if A and B are mutually exclusive, (2 marks)
(b) $P(B)$ if A and B are independent. (3 marks)
- (ii) Bag X contains 7 balls, of which 2 are red, 4 are green and 1 is blue. Bag Y contains 6 balls, of which 1 is red, 2 are green and 3 are blue. A biased die is thrown and the probability of obtaining an even is $\frac{3}{5}$. If the score on the die is even, Bag X is chosen, otherwise Bag Y is chosen. A ball is selected at random from a bag.
(c) Draw a tree diagram showing all the possible outcomes. (2 marks)
Hence, find the probability that the ball is
(d) red, (2 marks)
(e) green. (2 marks)
Given that the ball is blue, find the probability that it is from Bag Y . (2 marks)
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