

0765 Pure Maths with Mechanics 1

GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2024

ADVANCED LEVEL

Centre Number	
Centre Name	
Candidate Number	
Candidate Name	

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTION PAPER

Duration: One and a Half Hours

INSTRUCTIONS TO CANDIDATES



Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed "Advanced Level – 0765 Pure Maths with Mechanics 1".

4. Fill in the information required in the spaces above.

5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil:

Candidate Number and Name, Centre Number and Name.

Take care that you do not crease or fold the answer sheet or make any marks on it other than those asked for in these instructions.

6. **Answer All questions.**

7. **Formulae Booklets and calculators are allowed.**

8. Each question has FOUR suggested answers: A, B, C and D. Decide on which answer is correct. Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square brackets for the answer you have chosen.

For example, if C is your correct answer, mark C as shown below:

[A] [B] ☒ [C] [D]

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9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet, using, where necessary, the blank spaces in the question booklet.
12. **At the end of the examination, the invigilator shall collect the answer sheet first then the question booklet after. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

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Turn Over

SECTION A: PURE MATHEMATICS



1. Given the functions $f, g: \mathbb{R} \rightarrow \mathbb{R}$, where

$$f(x) = x^2 - 1 \text{ and } g(x) = 2x + 1,$$

$$(g \circ f)(2) =$$

- A 11
B 7
C 24
D 13

2. If $\log_x y = 2$ and $xy = 125$, then the values of x and y are respectively

- A 3 and 9
B 9 and 3
C 5 and 25
D 25 and 5

3. The vertical asymptotes of the curve

$$y = \frac{1}{(x-2)(x+3)}$$

- A $x = 3, x = -2$
B $x = 2, x = -3$
C $x = -3, x = -2$
D $x = 2, x = 3$

4. When the polynomial $p(x) = ax^3 + 5x^2 + 2x - 8$ is divided by $(x - 1)$, the remainder is -3 .

$$a =$$

- A -4
B -2
C 2
D 3

5. The set of values of x for which $\frac{(x+1)^2+3}{(x+3)(x-2)} < 0$ is

- A $\{x : x < -3 \text{ or } x > 2\}$
B $\{x : x \leq -3 \text{ or } x > 2\}$
C $\{x : -3 \leq x < 2\}$
D $\{x : -3 < x < 2\}$

6. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 4\beta + 4x, & \text{if } x < 1 \\ 4x - \beta^2 - 4 & \text{if } x > 1. \end{cases}$$

If $f(x)$ has a limit at $x = 1$, the value of β is

- A 2
B 1
C 3
D -2

7. Let p and q be statements.

p	q	$\oplus$
T	T	T
T	F	T
F	T	T
F	F	F

The compound statement $\oplus$ on the table could represent

- A $p \rightarrow q$
B $p \leftrightarrow q$
C $p \vee q$
D $p \wedge q$

8. Given the lines

$$L_1: \mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \alpha(4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$L_2: \mathbf{r} = \mathbf{i} + 3\mathbf{j} + \mathbf{k} + \delta(8\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$$

$$L_3: \mathbf{r} = 2\mathbf{i} + 6\mathbf{j} + 2\mathbf{k} + \beta(\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})$$

$$L_4: \mathbf{r} = 4\mathbf{i} + 12\mathbf{j} + 4\mathbf{k} - \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$$

Which pair of lines are parallel?

- A L_1 and L_2
B L_1 and L_4
C L_2 and L_4
D L_2 and L_3

9. A relation R defined on a non-empty set S is called a partial order if R is

- A Reflexive, symmetric and anti-symmetric
B Reflexive, anti-symmetric and transitive
C Reflexive, symmetric and transitive
D Symmetric, anti-symmetric and transitive

- 10.

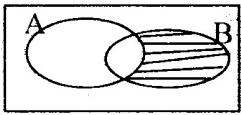
$$\text{If } \sum_{r=1}^{\infty} \frac{m}{2^{r-1}} = 10, \text{ then } m =$$

- A 10
B 5
C 15
D 7

11. The first order differential equation obtained from the function $y = Ax + \ln x$ is

- A $x \frac{dy}{dx} = y + \ln x + 1$
B $\frac{dy}{dx} = y - \ln x + 1$
C $x \frac{dy}{dx} = y - \ln(x + 1)$
D $x \frac{dy}{dx} = y - \ln x + 1$

12. The shaded region represents



- A $A \cap B$
 B $A \cap B'$
 C $A' \cap B$
 D $(A \cup B)'$

13. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by

$$f(x) = \frac{x+6}{x^2 - 2x - 15}$$

The domain of f is

- A $\mathbb{R} - \{3, 5\}$
 B $\mathbb{R} - \{-3, 5\}$
 C $\mathbb{R} - \{-5, 3\}$
 D $\mathbb{R} - \{-5, -3\}$

14. A curve is defined parametrically by

$$x = 2t, y = \frac{2}{t}$$

The Cartesian equation of the curve is

- A $xy = 4$
 B $xy = 2$
 C $xy = 1$
 D $xy = 3$

15. If $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{s} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, then

$\mathbf{r} \times \mathbf{s}$ is

- A $-4\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$
 B $-4\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$
 C $-4\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$
 D $-4\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$

16. Given that $f(\theta) = \sin \theta - \sqrt{3} \cos \theta$. Expressing $f(\theta)$ in the form $R \sin(\theta - \alpha)$ where $R > 0$ and α an acute angle, we obtain $f(\theta) =$

- A $2 \sin(\theta - \frac{\pi}{2})$
 B $\sqrt{10} \sin(\theta - \frac{\pi}{3})$
 C $2 \sin(\theta - \frac{\pi}{3})$
 D $\sqrt{10} \sin(\theta - \frac{\pi}{4})$

17. If $f(x) = x^2 + kx - 3$ has a root in the interval $[1, 2]$ then the range of k is

- A $k < -\frac{1}{2}$
 B $k > 2$
 C $-\frac{1}{2} < k < 2$
 D $k < -\frac{1}{2}$ or $k > 2$



18. Given that the sum and product of roots of the equation $2x^2 - 2x + q = 0$ are equal, the value of q is

- A -2
 B $\frac{1}{2}$
 C 2
 D $-\frac{1}{2}$

19. The sum of the first n terms of an arithmetic progression is $n(2n + 3)$. The n^{th} term of the progression is

- A $6n + 1$
 B $2n + 1$
 C $4n - 1$
 D $4n + 1$

20. $\left| \frac{-5+10i}{1+2i} \right| =$

- A 5
 B $\sqrt{5}$
 C $\sqrt{125}$
 D 6

21. The line L has equation

$\mathbf{r} = 5\mathbf{i} - 8\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ and the plane π has equation $2x + 2y - z = 5$. The sine of the acute angle between the line and the plane is

- A $\frac{3\sqrt{14}}{7}$
 B $\frac{\sqrt{14}}{7}$
 C $\frac{2\sqrt{14}}{7}$
 D $\frac{4\sqrt{14}}{7}$

Turn Over

22. $\frac{d}{dx}(3x + 2)^4 =$

- A $12x(3x + 2)^3$
- B $12(3x + 2)^4$
- C $12(3x + 2)^3$
- D $\frac{4}{3}(3x + 2)^3$



23. The equation of the circle which passes through the origin and the points (2,0) and (0,-4) is

- A $x^2 + y^2 - 2x + 4y = 0$
- B $x^2 + y^2 - 2x - 4y = 0$
- C $x^2 + y^2 + 2x - 4y = 0$
- D $x^2 + y^2 + 2x + 4y = 0$

24. The area of the finite region bounded by the curve $y = 2x - \frac{1}{x}$ and the lines $x = 1$ and $x = 3$ is

- A $8 - \ln 3$
- B $8 + \ln 3$
- C $8 - \ln 2$
- D $8 + \ln 2$

25. If a root of the equation $x^2 + 2x + 1 = 0$ is also a root of the equation $x^3 - 2x^2 + 6x + k = 0$, the value of the constant k is

- A 9
- B -9
- C 5
- D -5

26. A periodic function f , of period 4, is defined by

$$f(x) = \begin{cases} x^2, & 0 \leq x < 2, \\ x + 2, & 2 \leq x < 4. \end{cases} \quad f(9) =$$

- A 1
- B 3
- C 11
- D 81

27. Two functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ are such that $f(x) = 2x - 1$ and $(f \circ g)(x) = x - 2$. Then $g(x) =$

- A $\frac{x-2}{2}$
- B $\frac{x-1}{2}$
- C $\frac{2x-1}{2}$
- D $\frac{x+1}{2}$

28. The equation of the tangent to the curve $y = 3x(2 - x^2)$ at the point (1, 3) is

- A $3x - y - 6 = 0$
- B $3x + y - 6 = 0$
- C $y - 3x - 6 = 0$
- D $3x - y + 6 = 0$

29. Simplifying $\frac{\sin 5x - \sin x}{\sin 4x - \sin 2x}$ gives

- A $2 \cos x$
- B $2 \cos 2x$
- C $2 \sec x$
- D $2 \sin x$

30. The number of arrangement of the letters of the word *ARREARS* is

- A 210
- B 420
- C 640
- D 840

31. In the expansion of $\sqrt{3+x}$, the range of values of x for which the complete expansion is valid is

- A $-3 \leq x \leq 3$
- B $-3 < x < 3$
- C $-1 < x < 1$
- D $-\frac{1}{3} < x < \frac{1}{3}$

32. Evaluating $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$ gives

- A $\frac{1}{3}$
- B $\frac{3}{2}$
- C $\frac{40}{27}$
- D $\frac{27}{3}$

33. $\lim_{x \rightarrow \infty} \left(\frac{4x^2 - 1}{2x^2 + 1} \right)$ is

- A 0
- B 1
- C 2
- D 3

34. $\int_1^4 \left(\frac{x+3}{x} \right) dx =$

- A $3 + 4 \ln 3$
- B $3 + 3 \ln 4$
- C $4 + 4 \ln 3$
- D $4 + 3 \ln 4$

35. If the matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ -1 & 3 & 4 \end{pmatrix}$, then determinant of

M is

- A -18
- B -8
- C 0
- D 18

SECTION B: MECHANICS

36. The position vector of a particle at time t seconds is $\mathbf{r}$, where $\mathbf{r} = [(t^2 + 1)\mathbf{i} + t^3\mathbf{j}]$ m. The velocity of the particle when $t = 2$ is

- A $(4\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1}$
- B $(4\mathbf{i} - 12\mathbf{j}) \text{ m s}^{-1}$
- C $(2\mathbf{i} + 12\mathbf{j}) \text{ m s}^{-1}$
- D $(4\mathbf{i} + 12\mathbf{j}) \text{ m s}^{-1}$

37. The extension a force of 20 N produces in a spring of natural length 5 m whose modulus of elasticity is 150 N is

- A $\frac{3}{2}$ m
- B $\frac{3}{4}$ m
- C $\frac{2}{3}$ m
- D $\frac{1}{2}$ m

38. The velocity of a particle A relative to another B is $(7\mathbf{i} + 5\mathbf{j}) \text{ m s}^{-1}$. Given that the velocity of A is $(8\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1}$. The velocity of B is

- A $(15\mathbf{i} + 11\mathbf{j}) \text{ m s}^{-1}$
- B $(-\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$
- C $(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$
- D $(15\mathbf{i} - 11\mathbf{j}) \text{ m s}^{-1}$

39. Given that A and B are events with $P(A) = \frac{1}{4}$,

$$P(A \cup B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{12}$$

$$P(A|B) =$$

- A $\frac{1}{2}$
- B $\frac{1}{3}$
- C $\frac{1}{6}$
- D $\frac{2}{3}$

40. A small sphere of mass 0.6 kg is moving on a horizontal table with a speed of 10 m s^{-1} when it collides with a fixed smooth wall perpendicular to the table surface. It rebounds with a speed of $\frac{15}{2} \text{ m s}^{-1}$. The coefficient of restitution between the sphere and the wall is

- A $\frac{4}{3}$
- B $\frac{5}{3}$
- C $\frac{4}{5}$
- D $\frac{3}{4}$



41. Three particles of masses 5 kg, 3 kg and m kg lie on the y -axis at the points $(0, 4)$, $(0, 2)$ and $(0, 5)$ respectively. The center of mass of the system is at the point $(0, 4)$. The value of m is

- A 5
- B 4
- C 6
- D 7

42. A car wheel of radius $\frac{1}{2}$ m is moving at a speed of $\frac{10}{3} \text{ m s}^{-1}$. Its angular speed in rad s^{-1} is

- A $\frac{20}{3}$
- B $\frac{5}{3}$
- C $\frac{16}{3}$
- D $\frac{25}{3}$

43. A car accelerates uniformly from a velocity of 10 m s^{-1} to 60 m s^{-1} in 6 seconds on a straight road. The distance it covered is

- A 150 m
- B 200 m
- C 210 m
- D 215 m

44. A box contains ten green and six white marbles. A marble is chosen at random, its colour noted and it is not replaced. This process is repeated once. The probability that the marbles chosen are of the same colour is

- A $\frac{17}{32}$
- B $\frac{1}{3}$
- C $\frac{1}{2}$
- D $\frac{13}{32}$

45. The engine of a car of mass 1,200 kg works at a constant rate of 69 kW up a road inclined at an angle $\sin^{-1}\left(\frac{1}{6}\right)$ to the horizontal. Given that the non-gravitational resistance to the motion of the car is 300 N and taking g as 10 m s^{-2} , the maximum speed of the car is

A 15 m s^{-1}
 B 25 m s^{-1}
 C 30 m s^{-1}
 D 22 m s^{-1}

46. The work done by a force $\mathbf{F} = (i - 2j - 4k) \text{ N}$ which moves its point of application from a point A with position vector $\mathbf{r}_A = (3i - 4j) \text{ m}$ to another point B with position vector $\mathbf{r}_B = (-i - 6k) \text{ m}$ is

A 47 J
 B 36 J
 C 12 J
 D -6 J

47. A particle decelerating to rest has its speed $v \text{ m s}^{-1}$ at time t seconds given by

$$v = 6t - \frac{1}{2}t^2.$$

The deceleration of the particle when $v = 10$ is

A 4 ms^{-2}
 B 10 ms^{-2}
 C -10 ms^{-2}
 D -4 ms^{-2}

48. A uniform ladder AB , of length $2a$, rests in limiting equilibrium with its top end against a rough vertical wall and its lower end on a rough horizontal floor. The forces acting on the ladder are as shown in Fig. 1.

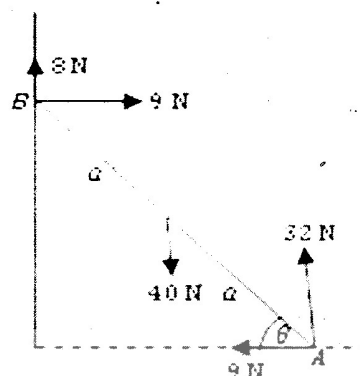


Fig. 2

The angle θ which AB makes with the floor is given by

A $\tan \theta = \frac{4}{3}$
 B $\tan \theta = \frac{3}{4}$
 C $\sin \theta = \frac{4}{3}$
 D $\sin \theta = \frac{3}{4}$

49. A particle of mass of 14 kg rests on smooth horizontal table. It is connected by a light inextensible string passing over a smooth pulley fixed at the edge of the table to a particle of mass 6 kg which hangs freely.

Given that the acceleration of the system is 3 m s^{-2} when it is released from rests, the force exerted by the string on the pulley is

A $42\sqrt{2} \text{ N}$
 B $82\sqrt{2} \text{ N}$
 C 42 N
 D 82 N

50. A smooth sphere A of mass 6 kg travelling at 4 m s^{-1} collides directly with a smooth sphere B of mass 3 kg travelling at 2 m s^{-1} in the opposite direction. Given that the speed of B after impact is 2 m s^{-1} , the kinetic energy of A after impact is

A 6 J
 B 12 J
 C 50 J
 D 13.5 J

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