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FURTHER MATHEMATICS PAPER 3
0775

WOURI COMMON MOCK EXAMINATION
GENERAL CERTIFICATE OF EDUCATION

March 2022

ADVANCED LEVEL

Subject Title	Further Mathematics
Paper No.	Paper 3
Subject Code No.	0775

Two and a half hours

Answer ALL questions

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets

Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

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..... TURNOVER

WOURI/0775/P3/MOCK 2022

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1. i) Three forces F_1 , F_2 and F_3 where $F_1 = (6i - 3j + 5k)N$, $F_2 = (3i + 2j - 3k)N$ and $F_3 = (-9i + j - 2k)N$ act through the points with position vectors r_1 , r_2 and r_3 respectively where $r_1 = (8i - 4j + 8k)m$, $r_2 = (-i + 4j - 5k)m$ and $r_3 = (11i - 2j + 5k)m$
- a) Prove that these forces are equivalent to a couple
- b) Find the magnitude of the couple (7 marks)

- ii) A body of mass 2kg is acted upon by a force $(2i - 4j + 6k)N$. Initially, the body is at the point with position vector $(i + 2j - k)m$ and moving with velocity vector $(3i - 4j - 2k)m/s$. Find the work done on the body during the first 4 seconds of its motion. (6 marks)

2. i) A particle P of unit mass moves on the curve with polar equation $r = a \sin \theta$ under the action of a force of magnitude $f(r)$, which is always directed towards the pole O. Show that
- a) $r^2 \dot{\theta} = h$ where h is a constant.
- b) $\dot{r}^2 = \frac{h^2}{r^4} (a^2 - r^2)$
- c) $\ddot{r} = \frac{h^2}{r^5} (r^2 - 2a^2)$
- d) $f(r) = \frac{2a^2 h^2}{r^5}$ (9 marks)

- ii) A particle P moves in a plane at time t seconds. Its polar coordinates (r, θ) are given by $r = at^2$, $\theta = \frac{1}{3}t^3$ where a is a constant.

- e) Show that the speed of P when $t = 2s$ is $4a\sqrt{17}$. (4 marks)

3. A smooth sphere A of mass 2m moving with velocity $(12i - 3j)m/s$ is struck so that it moves with velocity $6j m/s$. Find
- a) The impulse given to A
- b) The coefficient of restitution during the impact.
- Sphere A then moves on to collide obliquely with sphere B of equal radius but of mass m which is moving with velocity $(8i - 4j)m/s$. (9 marks)
- c) Find the velocities of A and B after impact, leaving your answers in terms of e, the coefficient of restitution between the spheres. (4 marks)

4. i) The rate of decay of a substance is cx where c is a constant and x is the amount of substance remaining after t seconds. Show that the time in which the amount of the substance is halved is $\frac{1}{c} \ln 2$. (5 marks)
- ii) A particle of mass m falls from rest in a medium in which the resistance is of magnitude $\frac{mgv^2}{h^2}$, where v is the speed and h is a positive constant. Find the distance it has fallen when the speed of the particle is $\frac{1}{2}h m/s$. (8 marks)
5. i) A light elastic string of natural length 2.4m and modulus of elasticity 15N is stretched between two points A and B 3m apart on a smooth horizontal surface. A body of mass 4kg attached to the midpoint of the string is pulled 10cm towards B and then released. Show that the subsequent motion is simple harmonic and find the speed of the body when it is 158cm from A. (8 marks)

ii) A particle P moves in a resisting medium such that its displacement x from a fixed point O is given by the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$. Show that P undergoes a damp harmonic motion and find the period of this motion. (5 marks)

6. Given the differential equation $\frac{dy}{dx} = x^2 + y^2$, find

a) a solution in ascending powers of x up to and including the terms in x^4 for which $y = 1$ when $x = 0$. (3 marks)

b) the value of y to 3 decimal places when $x = 0.1$ (2 marks)

c) the first 4 terms in the expansion of y as a series in ascending powers of $(x - 0.1)^3$, giving the coefficient to 3 decimal places.

Using (c), find the value of y to 3 decimal places when $x = 0.2$. Hence, using Simpson's rule, estimate to 2 decimal places the value of $\int_0^{0.2} y dx$. (8 marks)

7. A farmer knows from experience that when taking cuttings from orange trees, only 15 in every 100 successfully take root.

a) In a batch of 10 randomly selected cuttings, find the probability that

i) none of the cuttings take root

ii) fewer than 3 of the cuttings take root. (4 marks)

b) Let n be the smallest number of cuttings which need to be examined before there is at least 95% chance that one or more of them would have taken root.

iii) Show that n satisfies $(0.85)^n \leq 0.05$

iv) Given that $(0.85)^n = 0.05$, find the value of n . (4 marks)

c) Using a suitable approximation, estimate the probability that fewer than 6 in a batch of 50 cuttings take root. (5 marks)

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8. Prove by integration that the moment of inertia of a uniform rod AB of length $2a$ and mass M , about an axis perpendicular to the rod and passing through one end is $\frac{4}{3}Ma^2$.

The rod is held almost vertical with the end A resting on a horizontal plane which is rough enough to prevent slipping and is then released.

Find the angular speed of the rod while it remains in contact with the plane and it makes an angle θ with the upward vertical.

Prove that the end A loses contact with the plane when $\cos\theta = \frac{1}{3}$. (13 marks)