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FURTHER MATHEMATICS PAPER 2
0775



March 2022

WOURI COMMON MOCK EXAMINATION
GENERAL CERTIFICATE OF EDUCATION

ADVANCED LEVEL

Subject Title	Further Mathematics
Paper No.	Paper 2
Subject Code No.	0775

Three Hours

Answer ALL 10 questions

For your guidance, the approximate mark allocation for parts of each question is indicated

Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

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..... TURNOVER

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1. a) Given the matrix $M = \begin{pmatrix} -3 & 3 & -1 \\ 1 & -3 & 2 \\ 2 & -1 & 0 \end{pmatrix}$.

Find M^{-1} . Hence, find the image of the line $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{-1}$ under the transformation whose matrix is M .
(6 marks)

b) Consider the mapping with transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by

$$T(x, y) = (2x - 3y, x + 2y)$$

Show that T is a linear transformation.
(4 marks)

2. a) Expand $\ln(1 + \sinh x)$ as a series of ascending powers of x up to and including the term in x^2 . Hence, show that $(1 + \sinh x)^{2/x} \cong e^2 \left(1 - x + \frac{x^2}{2}\right)$.
(6 marks)

b) Find the polar coordinate of the point at which the cardioids $r = a(1 + \cos \theta)$ has tangent line which is parallel to the initial line.
(4 marks)

3. a) Given that $f(x) = \frac{2}{(1+x)(4+x^2)}$, express $f(x)$ in partial fraction and hence show that

$$\int_0^1 f(x) dx = \frac{1}{5} (26.6 - \ln 5)$$

(5 marks)

b) Show that $\int x^n \tanh^{-1} x dx = \frac{1}{n+1} x^{n+1} \tanh^{-1} x - \frac{1}{n+1} \int \frac{x^{n+1}}{1-x^2} dx$. Hence, evaluate $\int_0^{1/2} x^6 \tanh^{-1} x dx$.
(5 marks)

4. a) Find the value of the real constant λ for which the differential equation $\lambda \frac{d^2 y}{dx^2} - (3 - 2\lambda) \frac{dy}{dx} + (\lambda - 2)y = 0$ has a solution of the form $y = e^{\beta i(A + Bx)}$ where A, B, β are real constants. State the value of β for this value of λ . Obtain the general solution of the differential equation $\lambda \frac{d^2 y}{dx^2} - (3 - 2\lambda) \frac{dy}{dx} + (\lambda - 2)y = 2x^2 + 3x - 7$ for the value of λ calculated above.
(7 marks)

b) Given that $y = \operatorname{sech}^{-1} x$, show that $y = \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right)$.
(3 marks)

5. a) Given that $Z = \frac{-4+3i}{2-\sqrt{3}i}$, express Z in the form $a + bi$, where $a, b \in \mathbb{R}$. Hence, find $|Z^2|$ and $\arg 2Z$.
(6 marks)

b) Prove that for real θ , $\frac{1+e^{i\theta}}{\operatorname{Re}(e^{i\theta/2})} = 2e^{i\theta/2}$
(4 marks)

6. a) Verify that $k(k+1)$ is an even number $\forall n \in \mathbb{Z}$. Hence, prove that if a and b are odd integers, then $(a^2 - b^2)$ is a multiple of 8
(5 marks)

b) Find the length of the arc of the curve $x = \ln t$, $2y = \left(1 + \frac{1}{t}\right)$ between the points $(0, 1)$ and $(\ln 2, \frac{5}{4})$
(5 marks)

7. a) Prove that if $a \equiv b(\text{mod } n)$ and $c \equiv d(\text{mod } n)$, then $ac \equiv bd(\text{mod } n) \quad \forall a, b, n \in \mathbb{Z}$

b) Find the general solution of the equation $147x + 56y = 21$ for $x, y \in \mathbb{Z}$.

(10 marks)

8. a) Prove that the set $S = \{1, 2, 4, 7, 8, 11, 13, 14\}$ forms a group under multiplication modulo 15. There are three subgroups each consisting of 1, 4 and two other elements of S . List these subgroups and prove that two of them are isomorphic. (7 marks)

b) Prove by induction that $\sum_{r=1}^n 4^r = \frac{4}{3}(4^n - 1)$, where n is a positive integer. (3marks)

9. The curve C_f of the function f is defined by

$$f(x) = x + 2 + \frac{4e^x}{e^x + 2}$$

a) State the domain of f (2marks)

b) Find

i) $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ (4marks)

ii) $\lim_{x \rightarrow +\infty} [f(x) - (x + 2)]$ (2marks)

c) State the asymptotes of C_f (2marks)

d) Find the turning point of C_f if any (2marks)

e) Establish a variation table of f (4marks)

f) Sketch the curve C_f (4marks)

10. Two sequences (u_n) and (v_n) for $n \in \mathbb{N}$ are defined as follows

$$\begin{cases} u_0 = 3 \\ u_{n+1} = \frac{1}{2}(u_n + v_n) \end{cases} \text{ and } \begin{cases} v_0 = 4 \\ v_{n+1} = \frac{1}{2}(u_{n+1} + v_n) \end{cases}$$

i) Calculate u_1, v_1, u_2 and v_2 (4marks)

ii) Another sequence (w_n) is defined by

$$w_n = v_n - u_n, \quad \forall n \in \mathbb{N}$$

Show that w_n is a convergent geometric sequence (3marks)

iii) Express w_n as a function of n and obtain its limits. (2marks)

iv) Study the sense of variation (monotony) of (u_n) and (v_n) . What can you deduce? (4marks)

v) Consider another sequence (t_n) defined by

$$t_n = \frac{u_n + 2v_n}{3} \quad \forall n \in \mathbb{N}$$

Show that (t_n) is a constant sequence (3marks)

vi) Hence, obtain the limit of the sequence (u_n) and (v_n) (4marks)

THE END